International Journal of Approximate Reasoning 138 (2021) 67–75

Contents lists available at ScienceDirect

International Journal of Approximate Reasoning

www.elsevier.com/locate/ijar

Editorial

A retrospective on Isaac Levi: June 30, 1930 – December 25, 2018

Teddy Seidenfeld

Philosophy and Statistics, Carnegie Mellon University, United States of America

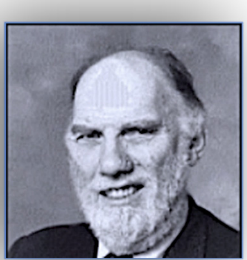
ARTICLE INFO

Article history:
Received 25 January 2021
Received in revised form 14 July 2021
Accepted 27 July 2021
Available online 8 August 2021

Keywords:
Belief revision
Direct inference
E-admissibility
Inductive logic
Indeterminate probabilities
Pragmatism

ABSTRACT

Isaac Levi's philosophy places him squarely within the tradition of American Pragmatism: the noble legacy of Peirce, James, and Dewey, evidently influenced by his teachers and colleagues at Columbia University, e.g., E. Nagel and S. Morgenbesser, and fellow graduate student at Columbia University, e.g., H.E. Kyburg, Jr. and F. Schick. Important for understanding Levi's original perspective on large scale philosophical problems is the theme that decision theory is embedded in them all. Typical of his work, Levi's contributions are grounded on significant distinctions, many of which are cast with the aid of sound decision-theory. In this retrospective I review four salient examples of his interests, spanning Levi's work on (1) *belief acceptance*, (2) *belief revision*, (3) *social philosophy*, and (4) *statistical inference*.



Isaac Levi's philosophy places him squarely within the tradition of American Pragmatism: the noble legacy of Peirce, James, and Dewey, evidently influenced by his teachers and colleagues at Columbia University, e.g., E. Nagel and S. Morgenbesser, and fellow graduate student at Columbia University, e.g., H.E. Kyburg, Jr. and F. Schick. Important for understanding Levi's original perspective on large scale philosophical problems is the theme that decision theory is embedded in them all. Typical of his work, Levi's contributions are grounded on significant distinctions, many of which are cast with the aid of

E-mail address: teddy@stat.cmu.edu.

https://doi.org/10.1016/j.ijar.2021.07.012
0888-613X/

sound decision-theory. In this retrospective I review four salient examples of his interests, spanning Levi's work on (1) *belief acceptance*, (2) *belief revision*, (3) *social philosophy*, and (4) *statistical inference*.[1]

First, however, it helps to know how he came to Philosophy and, for the focus of this retrospective, to understand the origins of Levi's distinctive approach – to use decision theory as a central tool in his philosophizing. Here is a bit of speculative rational construction I offer for that purpose.[2]

0. Early years

Levi's parents were Canadians by birth. (Their parents had emigrated from Lithuania and Galicia.) Levi's father was a Rabbi, who trained at the Jewish Theological Seminary in New York City. That is where Isaac was born (June 30, 1930). The family moved frequently, as his father was a somewhat itinerant Rabbi: Birmingham, Alabama; Auburn, New York; and in 1941 the family moved to Sydney, Australia, prior to the Pearl Harbor attack. In 1942, still in Australia, Levi's father joined the US Army, becoming the first overseas Jewish chaplain. Later in 1942, the rest of the family – Isaac, with his younger brother and sister, and mother – left Australia to stay with their maternal grandparents in Ontario, Canada. The family reunited, back in the Southern US, when his father, still a chaplain, returned from overseas in 1944 to be stationed in Hot Springs, Arkansas. (Isaac's lifelong fondness for the music of Hank Williams traces to this period.) After the war's end, the family moved to Detroit, Michigan, where Isaac graduated High School in 1947.

Isaac had a greater commitment to Jewish *religion* than to Jewish *culture*. This matched his atypical personal experience living as a child in small towns in Canada, the US South, and Australia, dominated by a (non-Jewish) Protestant culture, but having a rabbi for a father. With the aspiration of following in his father's footsteps, he attended college at New York University and entered the Jewish Theological Seminary in preparation for the rabbinate. During his first year of studies at the Seminary he won a scholarship which he shared with another student, Judith Rubins. They married (on Christmas Day) in 1951 and celebrated their 67th anniversary together on his last day. Together, they raised two sons, each successful in the arts and letters, and each married. Judy and Isaac celebrated three grandchildren.

When he entered the Seminary in New York, after High School, Isaac's religious views had been grounded on a philosophical position that the only rationale for (Jewish) monotheism – the belief in a single, unified deity – was that such a fact served as a basis for morality. But his exposure to Philosophy at NYU, in particular, what he learned from the critics of ontological arguments for the existence of God – what he learned from formidable teachers such as Paul Edwards and Sidney Hook – was the important lesson that "ought" does not follow from "is"!

Even if reason alone could establish existence of the deity, that argument does not entail a <u>*normative*</u> code of (Jewish) ethics. He realized that his religious views were dependent, not solely on religious facts – dependent not alone on the existence of a deity – but dependent also on *value judgments*, which are needed in order to support normative judgments of what one *ought* to do. It is this awakening, I speculate, that kept him alert throughout his career identifying implicit value-theoretic aspects tacit for sound methodology. It kept him alert to the role of utilities, and not merely coherent degrees of belief, in sound scientific practice. And, I speculate, this separation of fact and value helps to explain such details in his work as Levi's use of a cross-product of a (convex) set of probabilities and a (convex) set of cardinal utilities: where degrees of belief and values combine independently in his rule of *E-admissibilty*. "Is" alone does not entail "ought"!

The upshot of his undergraduate awakening was a conversion from religious monotheism to a then popular blend of positivism and pragmatism as a basis for underpinning morality; a position encouraged by others at the Seminary. These themes pointed Isaac to Philosophy at Columbia University and the thinking of John Dewey, who was the dean of American Pragmatism during the first half of the 20th Century. In the 1950s, Dewey's successor at Columbia was the acclaimed Philosopher of Science, Ernest Nagel, who served as the first John Dewey Professor of Philosophy at Columbia. Nagel's prominent students from that period included, in addition to Levi, Patrick Suppes, Henry Kyburg, and Frederic Schick.

Isaac's Ph.D. studies at Columbia (1951-57) moved him away from foundationalist aspects of logical positivism and towards non-foundationalist pragmatism. One important example that Nagel was fond of using when teaching Philosophy of Science, is that *observation reports*, especially as they appear in science, are *theory-laden* and not *theory-free*. Scientific observations incorporate consequences of an agent's volitions, e.g., acceptance of settled background assumptions, and are not merely a by-product of a passive spectator sport.

As Sidney Morgenbesser (the 2nd John Dewey Philosophy Professor at Columbia) emphasized, in order to know which inferences are legitimate it is important to identify the specifics that constitute the context of an inquiry – *What is the question*? For Levi, the question at hand helps to fix cognitive values that constrain the inquiry. For instance, fixing the question allows the investigator to distinguish relevant from irrelevant components of a potential answer – as reflected in the informational value afforded by a potential answer. In the Section 1, I discuss this theme in connection with Levi's theory of acceptance.

Levi's first full-time appointment was at Case Western Reserve in Cleveland, where he taught from 1957 until he moved to Columbia, in Fall 1970. He remained at Columbia for the rest of his academic career, where he became the 3rd John Dewey Professor of Philosophy – succeeding Nagel and Morgenbesser in that chair.

1 I thank two anonymous readers for their precise and constructive suggestions that prompted significant improvements.
2 See his "Self-Profile" in *Kyburg-Levi* [1], which serves as the basis for some of my speculative reconstruction, here.

This bio-sketch serves as a prologue to the following sampling of four of Levi's distinctive contributions.

1. Acceptance as a cognitive decision

In two early works [7] (1960) *Must the Scientist Make Value Judgments?* and [8] (1962) *On the Seriousness of Mistakes*, Levi argues (contra R. Rudner) that the values reflected in statistical type-1 and type-2 errors may formalize distinct cognitive, scientific values that are not to be conflated with economic, ethical, or political values. *Belief acceptance* – a voluntary act to adopt a new, full belief B in answer to a well posed "*which*?" question – is Levi's account of how to apply common standards of rational choice in the context of expected *cognitive* utility decisions. One engages in risky epistemic business when accepting a new full belief B – where B contains new, relevant information for the agent. B is one potential answer to the *which*-question. Prior to accepting B, the agent understands B might be false. That is the core philosophic idea in his (1967) book, *Gambling with Truth*, which formalizes the decision-theoretic tradeoff between making an error and acquiring an informative, true belief.

In more detail, the structural assumptions in *Gambling with Truth* require a (finite) *which-H* question, that Levi identifies with what he calls an *Ultimate Partition*: $H = \{h_1, \ldots, h_k\}$. The elements of H are the logically *strongest relevant* answers to the question that is of interest to the agent. This is a cognitive *value judgment*: finer partitions than H do not add *relevant* information, and coarser partitions than H lose *relevant* information regarding the question at hand as the agent identified that: namely, *Which element of H*?

The algebra $\mathcal{A}_H$ generated by H constitutes the set of possible relevant answers to the *which-H* question. In *Gambling*, Levi uses a (precise) credal probability Q and a (precise) cognitive, epistemic utility each defined over possible answers, $A \in \mathcal{A}_H$. Levi's novelty is in these utility functions. His idea is that the cognitive utility in accepting A as the strongest answer to the *which-H* question is a convex combination of two epistemic goals:

(i) an information function, the *content* (A),[3]
and
(ii) the truth of A, the indicator function $I(A)$.

The allowed tradeoffs between these two is required to be *truth-valuing* in that, in every state, a true answer A is preferred to a false answer A', regardless the relative content of the two answers A and A'. In *Gambling* Levi uses a uniform content function, where *content* $(h_i) = (k-1)/k$ for $i = 1, \ldots, k$.

With this machinery in place, the agent chooses an answer to the *which-H* question in accord with the decision rule to maximize epistemic (truth-valuing) utility. Levi adds a lexicographic consideration that favors suspending judgment among answers that have the same expected epistemic utility. That is, when A_1 and A_2 maximize epistemic utility, then so too does their disjunction, $A_3 = (A_1 \text{ or } A_2)$, which is favored by the lexicographic tie-break rule.

The upshot is an elegant acceptance rule. The allowed trade-off between goals (i) and (ii), above, yields an index of boldness, $0 \leq b \leq 1$ that operates as follows. Form a rejection set $R = \{h \in H : Q(h) < b/k\}$. In answer to the which-H question, with boldness index b, accept as a full belief the proposition B that is the disjunction of the *unrejected* elements of H. Evidently, this yields a well-defined, consistent extension of the agent's full beliefs – an expansion of her/his corpus of knowledge.

There are several notable features to this acceptance rule. Most important, it is not a function of the agent's credences, Q, alone. Acceptance depends upon the agent's cognitive values, through the *content* function and index of boldness, b. Also, under this rule a high probability is neither necessary nor sufficient for a relevant answer (i.e., for an element of $\mathcal{A}_H$) to be accepted. If $b = 0$, then only suspension of judgment (the disjunction of all the elements of H) is accepted. If $b = 1$ then all elements of H that are less probable than under the uniform distribution are rejected. That might result in coming to a full belief in a single element h^* of H, despite the fact that $Q(h^*) = \varepsilon + 1/k$ for some arbitrarily small $\varepsilon > 0$. That contrasts sharply with high-probability acceptance rules as proposed by, e.g., H.E. Kyburg.

Levi's belief acceptance rule is not the only mechanism he provides for expanding one's corpus of knowledge. In addition to *deliberate expansion*, which is how he identifies his acceptance rule, also Levi makes room for *routine expansion* of one's corpus of knowledge.[4] That is where the agent employs a (normal form) strategy to add propositions when a suitable procedure is followed. The routine is assessed *ex ante*, when the commitment is made. For example, one might assess ordinary perception as a sufficiently reliable source of truthful information that one commits to the routine of accepting ordinary observation reports. Then, seeing is believing, quite literally.

Routine expansion differs from *deliberate expansion* in that with the former the agent commits in advance of its application to following the rule for expanding one's full beliefs. With deliberate expansion, the agent assesses the context of the particular inquiry: forms a *which*-question, identifies a content function and boldness index, etc. before deciding what proposition to accept.[5]

[3] Levi requires that content $(A) = 1 - M(\mathrm{A})$ for some information determining probability measure M on $\mathcal{A}_H$.

[4] See chapter 2 of *The Enterprise of Knowledge* [12].

[5] Contrast Levi's routine and deliberate expansion with Kahneman's [5] fast and slow thinking.

2. Fallible versus corrigible full belief

But this epistemic story is sorely lacking if there is no guidance about how to correct error. The agent may create a contradiction in her/his corpus by a routine expansion. For remedying that, Levi distinguishes between the *certainty* and the *corrigibility* of a full belief. Chapter 1-3 in Levi's 1980 book, *The Enterprise of Knowledge*, emphasizes the importance of this distinction.

A rational agent's full beliefs serve as an epistemic resource by contributing to the framing of serious possibilities: those possibilities that are relevant to the agent's decision making of all stripes – both in practical and cognitive decision problems. In that sense a full belief in proposition B is *infallible*: B is taken as certain – there is no serious possibility for the agent that 'B' is false. Nonetheless, the rational agent recognizes that full beliefs are *corrigible*. The rational agent may change her/his mind and subsequently suspend judgment about B when properly motivated by other cognitive goals. See Chapter 2 of his 2004 [21] book, *Mild Contraction*.

Here is an interesting application of the distinction between *certainty* and *corrigibility* of full belief, relating to the Deweyan theme that knowledge acquisition is a social endeavor. (See Section 2.5, *Consensus-Based Ramsey Tests*, in Levi's 1996 book, [18] *For the Sake of the Argument*.) In [27] the *Fixation of Belief*, Peirce criticizes the method of *tenacity* because, among other defects, it fails to provide the way forward when opinions are in conflict. Agent$_1$ has full belief in proposition B_1. Agent$_2$ has full belief in proposition B_2. B_1 and B_2 are contraries. But the agents respect the opinion of the other. How shall they resolve this disagreement?

Because full beliefs are *corrigible* in Levi's approach, then without introducing error but by suffering a loss of information, each agent may contract her/his corpus of certainties to an epistemically neutral, informatively weaker position that suspends judgment between B_1 and B_2. That neutral position leaves open the question which of B_1 and B_2 is true. Then, the two investigators can carry forward jointly from this neutral position with an inquiry whether to accept B_1 or to accept B_2, or to continue in a state of suspense, using fresh experimental evidence to resolve the question scientifically.

The motivation for each to proceed this way, for each initially to suffer a loss of information, is their shared value: a respect for the other's opinion even when the other's belief is judged (certainly) false. If, instead, either had a different value and lacked respect for the opinion of the other on the subject of their dispute, there would be no reason for that agent to contract her/his beliefs to the neutral position.

Here, it is important also to distinguish between full belief as a *resource in inquiry* and full belief as *the outcome of inquiry*. For the latter to occur, the belief would first have to become doubtworthy. But not all reasonable full beliefs used as a resource in inquiry need be the outcome of inquiry. Levi [13, p. 29] gives a characteristic Morgenbesser counterexample.

> "How many of us have acquired the conviction that we have mothers through inquiry? How many of us ever had good reason to doubt that assumption and engage in inquiry concerning its merits?"

3. Social agents

In a 1982 essay, *Conflict and Social Agency*, Levi advocates for recognition of social agents. But the common economic view is that a social agent – thought of as a corporate entity composed of individual agents – cannot satisfy the same standards of economic rationality as that is required of an individual agent. One can read the second half of Savage's (1954) classic, [28] *The Foundations of Statistics*, as an attempt to find a suitable weakening of his theory of individual rationality that could serve to ground then-contemporary statistical practice as statistical decision making by a group of investigators. In his important [10] essay, *On Indeterminate Probabilities*, and in greater detail in his [12] book, *The Enterprise of Knowledge*, Levi shows that there are multiple aspects of uncertainty that, when distinguished, afford a uniform standard of rationality that applies both to individual and social agents. Here are some details and an illustration.

Canonical Bayesianism and its concomitant Expected Utility decision theory uses determinate uncertainty (a single credence function) and determinate valuation (a single cardinal utility) to represent an agent in decision making. That is the framework Levi uses in his [9] *Gambling with Truth*. But, as illustrated by the Ellsberg and Allais paradoxes, and as is well known to the members of SIPTA, there is also (respectively) a sense of *indeterminate* uncertainty and *indeterminate* value – where the decision maker's credences and values are represented by a non-trivial (convex) set of probabilities and a non-trivial (convex) set of cardinal utilities. Then, for example, a *neutral* group agent, formed by two individuals whose individual credences and values are different, so that these are conflicted, may be represented as the (convex closures of the) union of their respective sets of probabilities and the union of their respective sets of cardinal utilities. This unifies the standards of rationality between individual and social agents.[6] Also, it avoids familiar impossibility results associated with, e.g., pooling rules, where there is no satisfactory canonical Bayesian group agent that preserves consensus judgments of two canonical Bayesian agents who have different credences and cardinal utilities.[7]

A detailed account of Levi's decision rule(s) for decision making with indeterminate probabilities and utilities is beyond the scope of this review. Nonetheless, his seminal contribution in my opinion is to advocate for *E-admissibility*, which is

[6] See chapters 8-12 of *Hard Choices* [15].

[7] See Levi [16] for his perspective on where Pareto considerations do and do not contribute to the group consensus.

an important generalization of Bayesian Expected Utility maximization. Let $\{\boldsymbol{P}, \boldsymbol{U}\}$ be the (convex) sets of probabilities and cardinal utilities that represent the *indeterminate agent*. An option $\boldsymbol{o}$ from a menu of options $\boldsymbol{O}$ is E-admissible for this decision maker if there is at least one determinate probability P in $\boldsymbol{P}$ and at least one determinate cardinal utility U in $\boldsymbol{U}$ where $\boldsymbol{o}$ maximizes P-expected U-utility with respect to the menu $\boldsymbol{O}$. When the agent is Bayesian (i.e., has a determinate $\boldsymbol{P}$ and a determinate $\boldsymbol{U}$) E-admissibility reduces to maximizing Expected Utility. But E-admissibility makes precise different senses in which context matters for the indeterminate agent, though these do not matter for the determinate agent. For instance, with E-admissibility allowed choices from a menu of multiple options does not reduce to pairwise comparisons across all pairs from that menu. Knowing what is E-admissible in choices between all pairs from a menu does not generally determine what is E-admissible from that menu. By contrast, the canonical Bayesian agent with determinate preferences has an account of choice from a menu that does reduce to binary comparisons across the menu. (Likewise for the IP-agent who uses Sen/Walley's Maximality rule.[8]) In this sense, the indeterminate agent who uses E-admissibility – e.g. the group agent – is more sensitive to context than is the determinate agent.

Levi was involved in four papers/presentations at the biennially SIPTA meetings.[9] At the first ISIPTA-99 he used his theory of Indeterminate Probabilities and Utilities to make an important distinction between *indeterminacy* and *imprecision* that, in my opinion, remains undervalued still.[10]

A credence function is subject to *imprecision* when it is incompletely elicited or only partially identified. Ordinary, familiar limitations in human abilities make imprecision inevitable. We may specify probability values only to some fixed number, e.g. 5 decimal places. Nonetheless, an imprecise credence function remains subject to the norms (the *commitments*) for a rational credence function. If the rounding to 5 decimal places creates incoherence, that is a normative failure.

By contrast, an indeterminate credence function is one that has different norms and commitments compared with canonical Bayesian theory. The rational agent with an indeterminate credence that is represented by a specific (convex) set $\boldsymbol{P}$ of probabilities is not suffering an incomplete elicitation. With an appeal to E-admissibility as the (normative) decision rule for use with indeterminate probabilities, we may operationalize the difference between indeterminate and imprecise probabilities. I offer the following example, which helps also to explain de Finetti's well-known opposition to IP theory.

In de Finetti's [2] theory, the rational agent is a canonical Bayesian who is committed to some determinate credence function – a real-valued, coherent *Prevision* for (bounded) random variables. However, de Finetti is well aware that the rational agent may fail to fully identify her/his credence. His *Fundamental Theorem on Previsions* [2, p. 212] addresses this *imprecision*. Here is a summary of that result.

Suppose an agent provides a set of coherent, determinate previsions, one prevision for each (bounded) variable in a set χ, and where each variable is defined with respect to a common measurable space $\langle\Omega, \mathcal{B}\rangle$. The agent has no imprecision, nor indeterminacy, about the elements of χ. And these previsions fix determinate commitments for previsions over the linear span of χ. Let Y be another $\mathcal{B}$-measurable random variable but not in χ. The agent has not yet identified a prevision for Y.

Let $\underline{A} = \{X : X(\omega) \leq Y(\omega)$ with X in the linear span of $\chi\}$ and let $\overline{A} = \{X : X(\omega) \geq Y(\omega)$ with X in the linear span of $\chi\}$. Fix $\underline{P}(Y) = \sup_{X\in\underline{A}} P(X)$ and $\overline{P}(Y) = \inf_{X\in\overline{A}} P(X)$. Then, relative to the agent's coherent prevision over χ, $P(Y)$, may be any real number from $\underline{P}(Y)$ to $\overline{P}(Y)$ and, upon fixing $P(Y)$ in this interval, $[\underline{P}(Y), \overline{P}(Y)]$ the resulting expanded set of previsions is coherent. Outside this interval, the enlarged set of previsions is incoherent.

But prior to determining the value $P(Y)$ the interval $[\underline{P}(Y), \overline{P}(Y)]$ is merely an *imprecise* and not an *indeterminate* interval of previsions for Y, for de Finetti's agent. That is, this agent is committed to making decisions in accord with a determinate prevision for Y. For instance, suppose that option o_1 has greater expected utility than option o_2 for $P(Y) < (\underline{P}(Y) + \overline{P}(Y))/2$; that o_2 has greater expected utility than o_1 for $P(Y) > (\underline{P}(Y) + \overline{P}(Y))/2$, and so o_1 is indifferent to o_2 for $P(Y) = (\underline{P}(Y) + \overline{P}(Y))/2$. In a pairwise choice between o_1 and o_2, if the de Finetti agent finds that each is admissible, then the agent's commitments determine that $P(Y) = (\underline{P}(Y) + \overline{P}(Y))/2$. To continue, suppose that with options o_3 and o_4, o_3 has greater expected utility than does o_4 when $P(Y) > .25$; that o_4 has greater expected utility when $P(Y) < .25$; and that o_3 is indifferent to o_4 when $P(Y) = .25$. Last, suppose that $(\underline{P}(Y) + \overline{P}(Y))/2 > .25$. Then for the de Finetti agent, if both o_1 and o_2 are admissible in a pairwise choice between them, then only o_3 is admissible in a pairwise choice with o_4.

The situation is different for the agent with indeterminate previsions for Y, with IP interval $[\underline{P}(Y), \overline{P}(Y)]$, and who uses E-admissibility as her/his decision rule. Then, though both o_1 and o_2 are E-admissible in a pairwise choice between them, also both o_3 and o_4 are E-admissible in a pairwise choice between them. The interval of prevision values for Y, $[\underline{P}(Y), \overline{P}(Y)]$, is merely imprecise, not indeterminate for the de Finetti agent, which illustrates the operational content of Levi's distinction.

[8] See Schervish et al. [29] for distinctions between E-admissibility and the Sen/Walley *Maximality* rule.

[9] Levi [20]; Schervish et al. [29]; Levi [22], and Levi [24].

[10] The distinction between *indeterminacy* and *imprecision* is a special case of Levi's distinction between *commitment* and *performance*. The latter covers such challenging issues as how to understand the failure of logical omniscience for human agents when proposing normative standards of rational choice. See, e.g., Section 2.1 of *The Fixation of Belief and Its Undoing* [17].

4. Statistical inference

Levi's treatment of *chance* (i.e., objective probability) makes him a pluralist regarding the semantics of mathematical probability: The mathematical theory of probability is used both in his theory of credence for an agent, and in his account of chance. In Levi's approach, each of (determinate) *credence* and *chance* is a disposition predicate involving probabilities.

Using ideas presented a 1964 collaboration with S. Morgenbesser, [26] *Belief and Dispositions*, in chapter 11 of his [12] book, Levi promotes an account where dispositions are complex *place-holders* that are tied to test-conditions. For *credence*, the test-conditions relate to decision making. The primary challenge in interpreting *chance* as a disposition is "in specifying the epistemological relation between chance and test-behavior" (p. 235). This approach puts great weight on an account of *Direct Inference* for interpreting *chance*.[11]

In *Direct Inference*, evidence in the form of a *chance* statement regulates *credence* about a statistical sample. As an elementary illustration, suppose a coin is *fair* when flipped by method M, i.e., suppose there is a chance $1/2$ that it lands Heads rather than Tails when flipped by method M. Given this chance statement as (total relevant) evidence and the supposition that the coin is flipped by method M, then *Direct Inference* requires that the agent has a determinate credence of $1/2$ that the outcome of the flip is Heads (respectively Tails).

The interaction between Levi's account of indeterminate credence combined with his analysis of Direct Inference, provides him with a standpoint from which to debate important rival theories of statistical inference. See Levi's analysis [12, chapter 16] of H.E. Kyburg's [6]] and A.P. Dempster's [3] reconstructions of Fisher's *fiducial probability*. Here is a sketch of Levi's approach to that topic.

Central to Levi's analysis of these two accounts of fiducial probability is Direct Inference applied to pivotal variables: random quantities with determinate chance distributions that are a function of an observed quantity and an unobserved statistical parameter. For instance, let variable X have a parameterized chance distribution that is normal, $X \sim \mathbf{N}(\theta, 1)$, with statistical parameter θ. Then, even though θ is unknown and "prior" credence about the parameter is indeterminate, the quantity $V = (X - \theta)$ is pivotal with a known, standard Normal chance distribution $V \sim \mathbf{N}(0, 1)$. So, by Direct Inference, though the agent has an indeterminate credence about θ, she/he has a determinate credence, e.g., of approximately .95 that $-2 \le V \le +2$.

Is there an IP model for this problem where the agent can treat the observed value, $X = x$, as irrelevant to this Direct Inference about V? If so, then the agent's IP conditional credence for the event $-2 \le V \le +2$, given $X = x$, also is determinate with value approximately .95. However, given $X = x$, $-2 \le V \le +2$ obtains *if and only if* $x - 2 \le \theta \le x + 2$. Then the credal irrelevance assumption relating to the pivotal variable entails a determinate conditional credence about θ, given $X = x$. All despite an indeterminate "prior" for θ.

This is Fisher's enigmatic *fiducial inference*, which gains support within each of Kyburg's and Dempster's theories of interval valued probability. Levi's analysis provides original criticism of their (respective) accounts of Direct Inference, showing where each account conflicts with Bayesian conditionalization even in cases with determinate credences.

Levi's critical assessment of Kyburg's interval-valued probability highlights an important theme that affects IP statistical inference quite generally. It exposes a core disagreement about the extent to which IP should generalize Bayesian theory. In the following example, taken from Levi's [11], Levi pinpoints how a central principle of Kyburg's *Epistemological Probability* conflicts with basic Bayesian theory.

In Kyburg's theory probability is defined by what an investigator knows about frequency information in different reference populations. Let A and B be predicates with finite (set) extensions. The interval valued frequency information that:

$$\%\ A's \text{ among } B\text{'s is in the interval } [l, u]$$

fixes an *Epistemological Probability* assertion of the form.

$$\textit{Epist.Prob}.\ A(t) = [l, u]$$

where 't' denotes an individual that is known to be an element of the reference population, B.

The epistemological challenge is how to reconcile competing frequency information about the percentage of A's in n-many different reference populations $\{B_1, \ldots, B_n\}$, where the investigator knows:

$$\%\ A \text{ among } B_j \text{ is in the interval } [l_j, u_j]$$

and where individual t is known to belong to each of the reference populations, B_j $(j = 1, \ldots, n)$.

- What is the *Epist. Prob.* of $A(t)$ with respect to this corpus of n-many interval valued frequency assertions?

In Kyburg's theory, the *Strength Rule* is central in resolving interval valued frequency information relative to different reference sets.

[11] Also, see Levi's [11] Direct Inference.

Heuristic Idea for the Strength Rule:
Generally, where one reference set is a known proper subset of another, $B_1 \subset B_2$, then the fact that $t \in B_1$ entails $t \in B_2$. So, by the *Total Evidence Principle*, in general priority goes to the frequency information from the narrower (logically more informative) reference set B_1,

% A among B_1 is in the interval $[l_1, u_1]$
over % A among B_2 is in the interval $[l_2, u_2]$.

However, the *Strength Rule* reverses this priority when both $l_1 < l_2$ *and* $u_2 < u_1$. Then the broader reference class, that individual t belongs to B_2, though logically weaker information than that t belongs to B_1, carries more informative frequency information about the percentage of A's than does the narrower reference set B_1

- The *Strength Rule* gives priority to the broader reference set, i.e., it gives priority to the logically <u>less informative condition</u>, when that reference set provides <u>more informative frequency information</u>.

Levi's analysis pinpoints how the *Strength Rule* conflicts with basic Bayesian theory. Here is his illustrative example, which serves as a template for an important general finding.

Let $\boldsymbol{K}$ denote an agent's corpus of relevant knowledge about an individual, Peterson. $\boldsymbol{K}$, contains the following three items of information:

(1) 90% of Swedes are Protestants.
(2) Either 85%, or 91%, or 95% of Swedish residents of Malmo are Protestants.
(3) Petersen is a Swedish resident of Malmo.

By the *Strength Rule*, relative to K, *Epist Prob*. "Peterson is a Protestant" is [.90, .90], as the narrower reference set – Swedish residents of Malmo – carries less precise frequency information about being Protestant.

Next, consider these three simple statistical hypotheses, three versions of (2)

(2.1) 85% of Swedish residents of Malmo are Protestants.
(2.2) 91% of Swedish residents of Malmo are Protestants.
(2.3) 95% of Swedish residents of Malmo are Protestants.

Relative to these three consistent expansions of $\boldsymbol{K}$ by simple stat. hypotheses, we have these statements of *determinate Epistemological Probability*:

$K + (2.1)$ The *Epist Prob*. "Peterson is a Protestant" =[.85,.85],
$K + (2.2)$ The *Epist Prob*. "Peterson is a Protestant" =[.91,.91], and
$K + (2.3)$ The *Epist Prob*. "Peterson is a Protestant" =[.95,.95],

But by the *Strength Rule*, relative to these two consistent expansions of $\boldsymbol{K}$, also we have:

$K + [(2.1) \text{ or } (2.3)]$ The *Epist Prob*. "Peterson is a Protestant" = [.90,.90],
$K + [(2.1) \text{ or } (2.2)]$ The *Epist Prob*. "Peterson is a Protestant" = [.90,.90],

But $\boldsymbol{K}$ is equivalent to $(\boldsymbol{K} + [(2.1) \text{ or } (2.2) \text{ or } (2.3)])$.
Recall that relative to $\boldsymbol{K}$, the *Epist Prob*. "Peterson is a Protestant" = [.90,.90].

- But there is no Bayes model – no unconditional/conditional probability – that agrees with these 6 *determinate Epistemological Probabilities*.

In this example, we see that the *Strength Rule* yields determinate *Epistemological Probabilities* that are inconsistent with core *Bayesian* theory. The inconsistency between these two affect a variety of familiar Bayesian statistical differences, as I discuss in [31]. As a general finding, and expressed in somewhat different terms, Levi's analysis establishes the following: Giving priority to IP Direct Inference so as to increase the statistical precision of the conclusion, e.g., choosing the reference set to avoid *Dilation*, is inconsistent with IP reasoning using Bayesian conditionalization.

5. Concluding remarks

There is much more to Levi's *Pragmatism*: a wealth of useful distinctions that are not touched in this summary. He is not shy about tackling some of the eternal, big problems in Philosophy where those intersect his program. For one such example, I point the reader to Levi's discussion of the old question of free-will versus foreknowledge of one's own choices. The problem was live for W. James in his [4] *Dilemma of Determinism*. James asks, can there be *chance* present in the world (which for James is a necessary condition for free choice) if the deity is omniscient?[12] F. Schick revitalizes the challenge in his [30], *Self-Knowledge, Uncertainty, and Choice*. Levi's version of the problem, along with his solution, is found in chapter 4 (*Choice and Foreknowledge*) of his [15] book, *Hard Choices*n and Chapter 2 of his 1997 book, [19] *The Covenant of Reason*. Sometimes he headlined his position with the bold assertion: *Deliberation does crowd out prediction* [23].

Levi's fourth and final presentation to SIPTA was at ISIPTA-09, where he gave a short tribute to his dear friend, and intellectual competitor, H.E. Kyburg, *Busting Bayes: Learning from Henry Kyburg*. Though on opposite sides of numerous IP-related issues, their mutual admiration never waned.

Here is a photo in front of their Alma Mater at the 1982 Columbia University commencement ceremony, with Kyburg (standing to the right) receiving the Butler Medal for Philosophy in Silver, presented to him by Levi (standing to the left). The previous winner, in 1980, was their common Ph.D. thesis advisor, Ernest Nagel.

I encourage the interested reader to spend time with one of Levi's volumes of collected papers. For an overview of his self-proclaimed pragmatism, read his 2012 book, [25] *Pragmatism and Inquiry*. If I may be allowed a favorite, I recommend his 1984 book, [14] *Decisions and Revisions*. Both in content and in style, it captures the person I knew well as an effective teacher and thesis advisor, a sympathetic critic, and a good friend. The book's subtitle is *Philosophical essays on knowledge and value*. The attentive reader will gain much of each.

References

[1] R.J. Bogdan (Ed.), Henry E. Kyburg, Jr. & Isaac Levi, Profiles Series, D. Reidel Pub. Co., Dordrecht, 1982.
[2] B. de Finetti, Probability Theory, vol. 1, John Wiley & Sons, London, 1974.
[3] A.P. Dempster, Upper and lower probabilities induced by a multivalued mapping, Ann. Math. Stat. 38 (1967) 325–339.
[4] W. James, The dilemma of determinism, in: The Will to Believe and Other Popular Essays in Philosophy, 1897, Longmans, Green, & Co., New York, 1884.
[5] D. Kahneman, Thinking, Fast and Slow, Farrar, Straus and Giroux, New York, 2011.
[6] H.E. Kyburg, Logical Foundations of Statistical Inference, Reidel, Dordrecht, 1974.
[7] I. Levi, Must the scientist make value judgments?, J. Philos. 57 (1960) 345–357.
[8] I. Levi, On the seriousness of mistakes, Philos. Sci. 29 (1962) 47–65.
[9] I. Levi, Gambling with Truth, Knopf, New York, 1967.
[10] I. Levi, On indeterminate probabilities, J. Philos. 71 (1974) 391–418.
[11] I. Levi, Direct inference, J. Philos. 74 (1977) 5–29.
[12] I. Levi, The Enterprise of Knowledge, MIT Press, Cambridge, Mass, 1980.
[13] I. Levi, Doubt, context, and inquiry, in: Leigh Cauman, Isaac Levi, Charles Parsons, Robert Sdchwartz (Eds.), How Many Questions?: Essays in Honor of Sidney Morgenbesser, Hackett Publishing, Indianapolis, Indiana, 1983, pp. 25–34.
[14] I. Levi, Decisions and Revisions, Cambridge Univ. Press, Cambridge, 1984.
[15] I. Levi, Hard Choices, Cambridge Univ. Press, Cambridge, 1986.
[16] I. Levi, Pareto unanimity and consensus, J. Philos. 87 (1990) 481–492.
[17] I. Levi, The Fixation of Belief and Its Undoing, Cambridge Univ. Press, Cambridge, 1991.
[18] I. Levi, For the Sake of the Argument, Cambridge Univ. Press, Cambridge, 1996.
[19] I. Levi, The Covenant of Reason, Cambridge Univ. Press, Cambridge, 1997.
[20] I. Levi, Imprecise and Indeterminate Probabilities. ISIPTA-99, 1999.
[21] I. Levi, Mild Contraction: Evaluating Loss of Information Due to Loss of Belief, Oxford Univ. Press, Oxford, 2004.
[22] I. Levi, Convexity and E-Admissibility in Rational Choice. ISIPTA-2005, Andy Warhol Museum, 2005.
[23] I. Levi, Deliberation does crowd out prediction, in: T. Rønnow-Rasmussen, B. Petersson, J. Josefsson, D. Egonssson (Eds.), Hommage à Wlodek: Philosophical Papers Dedicated to Wlodek Rabinowicz, 2007.
[24] I. Levi, Busting Bayes: learning from Henry Kyburg, in: ISIPTA-09: Comments at Memorial Session for H.E. Kyburg, Jr., 2009.

[12] See James' long footnote 2 [4, p. 181].

[25] I. Levi, Pragmatism and Inquiry, Oxford Univ. Press, Oxford, 2012.
[26] I. Levi, S. Morgenbesser, Belief and dispositions, Am. Philos. Q. 1 (1964) 221–232.
[27] C.S. Peirce, The fixation of belief, Pop. Sci. 12 (1877) 1–15.
[28] L.J. Savage, The Foundations of Statistics, John Wiley & Sons, Inc., New York, 1954.
[29] M.J. Schervish, T. Seidenfeld, J.B. Kadane, I. Levi, Extensions of expected utility theory and some limitations of pairwise comparisons, in: ISIPTA-03, 2003.
[30] F. Schick, Self-knowledge, uncertainty, and choice, Br. J. Philos. Sci. 30 (1979) 235–252.
[31] T. Seidenfeld, Forbidden fruit: when epistemological probability may not take a bite of the Bayesian apple, in: E. Kyburg, W. Harper Jr., G. Wheeler (Eds.), Probability and Inference: Essays in Honor of Henry, College Publications, London, 2007, pp. 267–279.